Citations and Diffusion of Knowledge: An Economic Analysis*

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Abstract

Citation patterns in many academic disciplines have displayed a pattern of connections similar to those observed in many other different real-world contexts, such as links on the world-wide web. The various models that have been proposed to generate such networks, generically called "preferential attachment models", rely solely on random link formation and copying and do not take into account rational choice among authors in an academic community, which would consider the competition for citations and ensuing professional success. In this paper we construct such a model with rational agents to understand some aspects of citation patterns and knowledge diffusion in a specific academic field .. We show that rivalry or competition in citations might be an obstacle to diffusion, depending on behavioral rules specific to the field. Increased heterogeneity in the quality of papers reduces this effect. After considering models with complete information, we analyse models with private information about quality of one's own paper and use the framework to consider the interaction of this process with acquaintance networks and strategic entry. Superimposing the citation process on an acquaintance network yields patterns different from preferential attachment. Strategic entry leads to cascades of papers. Though we might have ex-ante efficiency in some equilibria, ex-post efficiency is not guaranteed. Ex post efficiency

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cannot be guaranteed since it is always possible in equilibrium that a good paper "dies" and a worse one survives, but ex ante efficiency is sometimes attainable.

1 Introduction

The aim of this paper is to model aspects of the process by which rational agents engaged in research use and cite earlier related work in their field. Citations constitute visible evidence of the diffusion of ideas and are therefore important in studying the influence or impact of particular pieces of research. It is also possible to think of citations as directed links in a network, so the nature of the diffusion of ideas also endogenously generates a network whose properties can be studied.

It is natural to think of citations in academic research, since the academic enterprise is one with which we, engaged in it, are intimately familiar. Academic administrators often ask for citation counts as evidence of the impact of an individual's research and this might translate into salary raises or external offers. Being cited by one's peers also gives us pleasure and not being cited, when one should have been, is frequently cause for discomfort. Citations therefore have real consequences for an academic's utility.

We can also think of citations of patents in industrial research and development in a similar way. An individual firm might have a project that could be facilitated by using someone else's idea. The firm could choose, however, to try to avoid having to pay royalty fees for using the patent and develop its own original solution to its problem. Such a new product or process could itself generate fees from future entrants to the field.

We shall focus on academics for convenience, though sometimes keeping the industrial R&D context in mind could help motivate some of the assumptions.

As stated earlier, we shall not consider every aspect of the decision on whether to cite a preceding paper. In particular, we abstract from issues relating to repeated interactions, where reciprocity could play an important role. The basic tradeoff that we shall examine is that between investigating and using an existing good idea to simplify one's own task versus expending costly effort to come up with one's own solution to generate high future payoffs from others with similar projects.

The context we wish to model might be thought of as a research field progressing by the solution of many related small problems. The person who solves one of these problems puts in effort and gets an expected value commensurate with the effort put in from her proposed solution to the problem. However, if other papers are known to exist in related areas, an individual might read one or more of them to obtain ideas that would simplify her task thereby reducing the cost of effort and increasing the value of the solution to her. Any ideas from other papers so used would need to be cited. (This is an assumption, but probably a good one for the vast majority of researchers; it is certainly something that firms engaged in R&D have to do, to avoid lawsuits) However, by citing a related paper, the individual who cites signals to other future entrants that the cited paper has proved useful and therefore directs other researchers to it rather than the academic's own work, whose usefulness to future entrants is uncertain. Note that in much of this paper, we assume that there is *perfect information about who has done relevant work*, though whether the relevant work is useful is unknown before someone investigates it. Survey papers and textbooks often garner citations because they themselves cite large numbers of other papers and books and serve to dispel lack of information about earlier research. With perfect information, this motive for citing (or for writing textbooks) is absent.

There appear to be relatively few papers dealing with the endogenous formation of citation networks. One exception is the paper by Mikhail V. Simkin and Vwani P. Roychowdhury ([23]), which relies on random copying of citations in previous papers.[1]In this model, an author randomly chooses some previous paper that appears related and randomly copies some proportion of the references in that paper. The randomness generates increasing returns; the more one is cited the more often one will be cited. However, here the number of citations of a paper is independent of that paper's characteristics, which appears to suggest that administrators counting citations are irrational. In the R&D context, random citation could lead to a high volume of lawsuits for patent infringement, though Simkin and RoyChowdhury do not intend their paper to apply to this.

As a contrast to our view of the sequence of related small research problems as constituting a field, the paper by Paul David [10] considers different scientists pondering the truth or falsity of some major proposition or theory. They become aware of the opinions of individuals they are connected to; since these opinions contain information, Bayesian updating leads to their adopting the opinions of the majority in their neighbourhood; the neighbourhood structure is *given*. The ultimate disposition of the theory is then found by using techniques from probability theory, namely the voter model discussed in Rick T. Durrett (1988). An individual cites the opinions of others in his neighbourhood as justification for his own opinion. This explanation has something in common with lawyers citing precedent and case law to justify a particular argument. One can interpret the neighbourhood here as coming from a social network. In a later section, we consider the interaction between the process we model and the existing social network to see how the latter constrains the former in situations where information about past work flows only through the social network (as in David's model).

Citation patterns have been used in empirical exploration of academic research commu-

[1] [9] has a sentence suggesting copying of citations might be rational: "Signalling by third parties: The latter, when deciding whom or what to cite, may be concerned to demonstrate that they are conversant with the reputational ranking of people in a specific area of science."

nities in physics by Sidney Redner([21]); C. Lee Giles and Isaac G. Councill ([14]) have used acknowledgements to trace a similar network of influence. These various networks display some form of a "power law" structure in aggregate; that is, the degrees of nodes in the network follow a power law distribution with a small number of highly connected nodes[2]

One early empirical discussion of citations is contained in Derek S. Price [20]. He looked at the patterns from 1862 to 1961 across many fields and did extensive analysis of the empirical regularities of the network of scientific papers. Price calculated that on average there were seven new papers a year for every hundred papers in that field, and each new paper contained about fifteen references. Therefore, on average, each paper is cited once per year. However,he found that in *any given year*, about 35% of existing papers are not cited at all and 49% are cited only once. Of the others, the percentage of papers cited n times falls off rapidly with n (in the order of $n^{2.5}$). The data appears to fit the hypothesis, according to Price, that about 10 percent of existing papers "die" each year. Also citations for a paper tend to occur in "capricious bursts".

The findings of Price regarding the rapidly decreasing proportion of papers with higher citations has the same flavour as the "power law" mentioned earlier. The models constructed to generate networks following such a power law, such as the preferential attachment models of Albert-Laszlo Barabasi and Reka Albert ([4]) and Bela Bollobas and Oliver Riordan ([5]), all rely on some exogenously specified process by which links are formed in the network.

In these models new nodes are born each period and each of them links with a existing one randomly but with a probability that is proportional to the number of links the node already has. This results in a well-defined stochastic process and we can calculate the properties of the network generated. Here, the older nodes would tend to have more links than newer ones and the process implies that there is a tendency of cumulation, which is similar to the observed cumulation of citations to a small set of papers.

In our model we will try to address the '*why*' of the preferential attachment model in the specific context of academic citations by pinning down the possible economic motivations at work.[3]

Our paper also relates to the literature of the spread of technology and information among agents in a community.[4]The results we obtain illustrate the effects of competition

[2]The "power law" states that the probability that a randomly selected node in a network has r links is $r^{-\gamma}$. The parameter γ has been estimated to be between 2 and 4.1 for different networks such as the web or the network of citations.

[3]The nature of citations bears some resemblance to that discussion of "cumulative advantage" presented by Robert K. Merton in the 'Matthew effect' papers. Small (2004) makes this connection clear-"When a paper is cited, other authors can see that it is and this heightens their interest in the paper and their likelihood of citing it as well at some later date. In this sense, citation acts like an expert referral."

[4]See [11] for economic models of such social dynamics and [6] for a discussion of social learning.

and strategic considerations on the diffusion of useful ideas. For efficient dissemination of ideas, existing papers should be investigated immediately to see if they have ideas that are broadly useful. To the extent that competition delays such investigation, it creates some inefficiency. In some variants of our model, such inefficiency occurs. We briefly summarize the qualitative features of our results below.

In the model with complete information we find an irreducible multiplicity of equilibria. However, the multiplicity is caused by different agents using different rules to identify past papers to investigate. If all agents use the same rule, corresponding to norms in different fields,[5]we obtain a single equilibrium for each rule. The different rules have different implications for how efficiently knowledge diffuses and give rise to different patterns of investigation and citation. In particular, the norm that specifies citing only the most recent paper in an area leads to inefficient delay and the pattern of the expected number of citations oscillates with the age of a paper, whilst for the other norms, earlier entrants (the pioneers) should expect to be cited more often. These results are with a finite number of entrants. With an infinite horizon, the unique stationary mixed strategy equilibrium does not display the oscillating pattern

We then assume each agent has a better idea about the usefulness of her own paper than other agents. The private information leads to a combination of behavioural norms-randomising among uncited individuals initially and then choosing the most recent previous entrant.

We then consider strategic entry if the agents all have their ideas simultaneously. We find there exists a "signalling" equilibrium, in which earlier entry implies higher average quality. Once entry occurs, there is a cascade of related papers.

Finally, we constrain the directed citation graph by an undirected acquaintance network. Now we relax the assumption that the existence of all previous papers is known and assume instead that one learns about papers that acquaintances have written or ones they have cited.[6]This gives rise to the closest analogue in our paper to the preferential attachment models. The probability a paper is cited then depends on two factors, one (the number of previous citations) arising from the (superposed) network structure and the other (which is a probability itself and hence less than 1) arising from the strategic incentives of players. The process is sublinear and therefore does not give a power law (see Fan Chung, Shirin Handjani and Doug Jungreis (2003)).

We make several strong assumptions in our model, though it is not clear that relaxing them would give any new insights. The two strongest ones are: (i) Once a previous piece of work is found useful by someone, it will be found useful by everyone following and if it

[5] hese are: citing the most recent paper or the oldest one or citing all available papers with equal probability

[6] This does not seem that far-fetched though whether reciprocity is at play here is hard to tell.

is not useful, it remains not useful; (ii) Only one citation is allowed per new paper. We discuss relaxing the first in the extensions section (section 7). The second implicitly takes into account the time spent in investigating past work or past patents. One could think of it as choosing to refer to one directly useful paper and referring to survey articles or textbooks for the others.

The rest of the paper is organized as follows. In Section 2 we describe the basic model while Section 3 deals with the analysis.In section 4 we introduce a model with private information about types. In Section 5 we discuss strategic entry decisions. Section 6 deals with social networks. Section 7 concludes with discussions on possible efficiency issues and on introducing heterogeneous quality . All detailed proofs are relegated to the appendices.

2 The benchmark model: single entrant per period and two qualities

The set of players is denoted by N={1,2,...k,...,n}. Players are ex ante identical. In each period, one player enters; the order of entry is predetermined. We shall denote by Player k the individual who enters and writes a paper in period k. Agent k can write on his own or use some existing paper, 1,2,...$k-1$ before publishing (or "entering"). A paper k can be "useful" or "not useful"[7]. If useful, the paper gives a value[8] $v > 0$ to any player $k+1, ...N$ who cites it. The payoff to the paper being cited is w for each citation it gets. We assume $v > w$. If not useful, the value is 0 and the paper is not cited. The prior probability that any paper k is useful is p_0.Paper k being useful is independent of the usefulness of the sequence of papers $1, 2...k-1$. Any entrant first observes the state of citations C_{ik}, the number of citations received by paper i till period k. We assume $C_{ik} \geq 1$,that is, writing a paper is, by convention, a citation. Any paper with $C_{ik} \geq 2$ is revealed to be useful. After entry each agent updates his beliefs about each agent/paper. Then he decides whether to incur cost c to *Investigate*(read) some $j = \{1, 2, ..., k-1\}$ or *not* to *investigate* at all (NI). We assume $p_0 v > c$. If investigated, it is revealed whether $j's$ paper is useful or not. If useful, agent k decides whether to use (and cite) it or not. At the time agent k enters, he observes the identities of all previous entrants and the citations each has, including the virtual self-citation. But k does not observe the *actions* prior entrants have taken with respect to reading or not reading previous papers. Therefore k is unable to distinguish between the histories where Player i $(1 < i < k)$ chooses NI and where she chooses I but does not cite (because, perhaps, the investigated paper was not useful). By choosing to cite(use the paper of a

[7]This simplification is made for analytical tractability-clearly papers can be useful to different degrees.

[8]This could be interpreted as the additional value obtained from a useful paper.

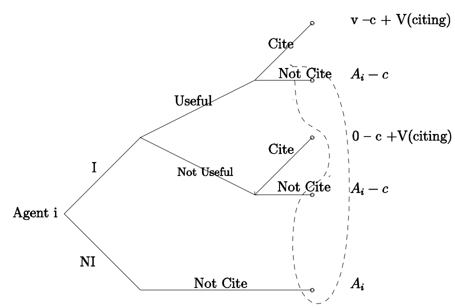

Figure 1: Original game

previous agent) agent k gets an immediate payoff of v and some future payoff depending on whether he is ever cited, which again depends on the state C_{k+1} at $t = k + 1$. By choosing NI or not cite, even after Investigation, he has to write on his own and gets a lower current payoff, which is normalised to 0, and an expected payoff depending on the state next period. After $k's$ decision, the next agent enters, observes state $C_{i,k+1}$ and takes decisions as specified in the previous steps. All agents have the same discount factor $\delta \in (0, 1)$.

We can represent the actions of any player i in a schematic tree(Fig 1) where A_i is the expected future benefit from not citing any previous player.

Note that after the uncertainty is resolved, the paper either gives a payoff of v or 0. As long as the cost of reading c is positive, an agent i would always use(cite) $j's$ paper if he found it useful after reading (otherwise it would have been optimal not to read it). Also, if not useful i has no choice but to write on his own as if he had not read any other paper in the first place. So, this game can be reduced to an equivalent game, represented by Fig 2.

Note that both the nodes t2 and t3 involve no citation and are in the same information set. Hence, if the equilibrium strategy of i is to I and he deviates, then $i + 1$ observes no citation and believes that i is at node t2, when he is actually at t3 and so not all deviations are detected.

Let us now consider the assumptions made in the specification of the model. The payoff v can be interpreted as a private benefit an agent gets from writing a paper. The payoff from a paper when it uses another's idea (v) is higher than when it is written solely by the agent, due to the fact that the agent writing a paper entirely on his own has to put in a much higher effort to achieve the same quality than when he is supplementing his idea with that of another agent. Hence the net benefit of writing a paper(not taking the cost into account)

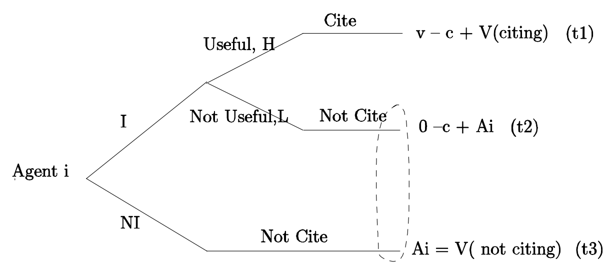

Figure 2: Reduced game

is lower if the agent writes by himself. The payoff w is the benefit from the fame and other associated consequences an agent gets from being cited.

The assumption made for analytical tractability is that a paper is either always useful (high quality) or never useful (low quality). First, note that the evolution of the probabilities is now very simple. Following one success of an investigation, the paper will be revealed to be of high quality and belief about its usefulness goes up from p_0 to 1. On the other hand, one failure does the opposite. If it is known that a paper was investigated but not cited, it is revealed that the paper is of low quality and it is never investigated again by any agent i.e. $p_s = 1$ and $p_f = 0$. In other words, we have perfect signals regarding the quality of the papers investigated. As for papers not investigated, the belief remains at the prior p_0.[9]

[9] In the basic model, the probability that a paper is useful in any one instance, given it is of high quality, is given by h and, given it is of low quality is given by l. For the sake of tractability we took $h = 1, l = 0$, which implied that once a paper is found useful (or not useful), it remains so for all future readers. If we relax one of the equalities then the tradeoff remains the same. For example, if $l = 0, h < 1$ then one citation of a paper would reveal that it is of high quality as in the basic model. A non-citation however does not reveal for sure that it will never be useful but the probability of its usefulness goes below the prior. Hence an agent would not investigate that paper. Similarly, for $l > 0, h = 1$ one non-citation reveals that it must be a low-quality paper and the probability of it being useful later is less than the prior whereas a citation increases the proabability of another sucess for that paper.

For general h and l, i.e. $h > l > 0$, we would get a non-degenerate distribution of citations. Bayesian updating would then imply that the prior p_0 goes up following one success but not to 1. Generally, it is increasing in the number of successes and decreasing in failures. For a given (h, l), agents follow the Bayesian updating rule and there will be a certain number of failures, x_f, of a paper after which the belief about its usefulness goes below p_0. So, we might observe patterns of the following sort: a paper is investigated for the first time and if it is a failure the belief goes below p_0 and it is not chosen again. If it is a success it gets a citation, the belief increases to $p_1 > p_0$ and it is chosen again. This second investigation might be a failure and hence the belief goes down to $p_2 < p_1$. However, p_2 might still be greater than p_0, in which

The trade-off involved here is between investigation of some $j's$ paper with potential current benefit v associated with lower future benefit (because by citing the agent i is signalling that $j's$ idea is useful) or no investigation (and hence no citation for sure), which has lower current payoff but a higher expected future payoff (since i does not give anything away about j). We want to focus on the equilibrium pattern of investigation and citation.

We also assume a version of private uncertainty in the basic model in that a particular agent does not know the quality of his own paper, that is, his belief about the probability that his paper would be useful to somebody is also p_0. This is sometimes reasonable in the context of academic papers since the quality of a paper is determined by the judgement of one's peers. Also, this implies that no agent knows his own type (high or low), which helps us to abstract initially from signalling motives.[10]

2.1 Strategies, Payoffs and Equilibrium

We define strategies for the N players and the equilibrium of the game.

Strategies: *Let the set of information sets where agent k has to move be IS_k and let z be an element of IS_k.A strategy s_k for the agent k is his choice of an element from the set $S^k = \{NI/I_0, I_1, I_2, ..., I_{k-1}\}$ at each z, given his entire set of beliefs $\mu^k = (\mu_1, \mu_2, ...\mu_{k-1})$ about the probability of usefulness of all earlier entrants at each z.Thus a strategy $s_k = \{(I_i^z)\}_{z=1}^{Z}$, where $|IS_k| = Z$ and $i = 0, 1, ..., k-1$.*

Agents choose their strategies to maximize their expected payoffs, where the expectation is with respect to the beliefs μ.

Equilibrium: *A Perfect Bayes Equilibrium is a N-tuple of strategies for all N players $< s_1^*, s_2^*, ...s_N^* >$ such that s_k^* is a best response to s_{-k}^* at every information set of Player k,given beliefs μ,which are derived from the prior p_0 and the history of play using Bayes' theorem, wherever possible.*

To derive the beliefs, note that, at any history h_t which has a citation other than a self-citation, it is revealed that the paper was investigated and is of high quality and will guarantee v if investigated. All papers with any citation will have this feature. Other papers

case, it is chosen again. Consecutive failures will take the belief below p_0. Once that happens this paper would not be chosen again and papers whose quality is at the prior level will be chosen and the same process followed. Hence, we might observe a group of papers with more than one citation, though the second paper was chosen only when the first cited paper had enough consecutive failures. In fact, we can also make h and l dependent on the number of citations of a paper. For example, when a paper has been used and cited say m times, the probability that there is anything useful remaining in it decreases, which implies that both h and l are decreasing in the number of citations a paper gets. This will also result in a distribution of citations instead of a spike for only one paper .

[10] We do discuss possible effects of signalling in Sections 5-8

without any citation can belong to one of the two groups: a) the paper has been investigated but not found useful, in which case $p = 0$ for sure; or b)the paper has not been investigated in which case the ex-ante probability of it yielding v is $p_0 < 1$. So, if an agent wants to choose I at any node, he will choose the paper with a citation since the expected payoff from so choosing is the highest. It follows that once there is a revelation of a high quality paper k at time t, that paper will be investigated (and cited) by all agents from time $t+1$. This is true since the person investigating paper k at time t, must have done so because the expected payoff from investigation is more than that from not investigating a paper with usefulness probability p_0. Given this, investigation of a surely high-quality paper must have a higher expected payoff for agent $t+1$ onwards. Moreover, once a paper is cited, any paper without a citation (including the current entrant's) ceases to be competitive and the trade-off disappears. So, for any history with a citation of paper k', the equilibrium strategies of subsequent entrants will be investigation of paper k' and consequent citation for all periods hence. The trade-off between present and future benefits mentioned is relevant only after a history with no citations.

Let h_t' denote any history with no successes (or equivalently citations). We have to specify equilibrium strategies (NI or I_k) for each agent k after such a history h_t'. We define an *equilibrium string* for this purpose.

An Equilibrium String is a N-dimensional array where the k^{th} element is the equilibrium decision of the k^{th} agent from set S_k after a history of no citations. Since the trade-off between current and future benefits kicks in at these histories, we have to figure out what the *equilibrium string* is, which along with the decision $s_k' = I_j$ *whenever* $\exists j$ *with* $C_j > 1$ and the belief μ, will be the Perfect Bayesian Equilibrium.

To completely specify the equilibrium we have to specify the belief μ_{ik} each agent k has about i, $i = 1, 2, ...k-1$. On the equilibrium path, μ is derived by Bayesian updating. Information regarding the paper is revealed following one citation and $\mu_{ik}(\mathrm{c}_i > 1) = 1$ while $\mu_{ik}(C_i = 1) = p_0$ if i was investigated with probability 0 in equilibrium. If i was investigated with some positive probability r in equilibrium, then $\mu_{ik}(C_i = 1) = (1-r)p_0 < p_0$. Histories off the equilibrium path involve deviations that *are* revealed to be such. If some i deviates from I_j to NI , this is not observable by $k > i$.If instead, i deviated from NI to citing some j not cited before, then this deviation might be observed if j is useful. The out-of-equilibrium belief here can naturally be set at $\mu_j = 1$. If the deviation is not revealed to $k > i$ then $\mu_{kj}(C_j = 1)$ remains at p_0 (this is on the equilibrium path).

3 Equilibrium Analysis

We first give a trivial lemma for the updated priors.

Lemma 1 *If some previous agent is investigated with positive probability, then in states of no citation, the belief regarding his usefulness is less than the prior* p_0.

After history h_t with some $C_{jt} > 1$, the equilibrium behaviour is also trivial and given by:

Lemma 2 *After any history* h_t *with* $C_{jt} > 1$ *for some* $j = 1, 2...t,$ *agent* $t+1$ *chooses to Investigate* $j^* = \arg\max C_{jt}$ *in equilibrium.*

Proof. We prove this by backward induction. Suppose at time t, agent t enters and observes $C_i > 1$(wlog).Consider the last agent, N. He chooses to investigate $\widehat{j}$ = argmax $C_j, j < N$, since he only cares about the current benefit. Now, suppose agents $\tau, \tau+1, ..., N$ follow this strategy. We need to show Player $\tau - 1$ also follows this strategy. (Note:If there is a $C_j > 1, j < \tau$, then argmax $C_j, j < \tau$ is same as argmax $C_j, j < \tau - 1$, since no agent except agent τ can cite $\tau - 1$, so this player cannot have more than a self-citation). He knows that τ will choose $\widehat{j}$ and hence, the expected future benefit of $\tau - 1$ is 0. Given some $C_{\widehat{j}} > 1$, $\tau - 1$ obtains a payoff $v - c$ from investigating $\widehat{j}$, $p_0 v - c$ from investigating j with $C_j = 1$ and 0 from not investigating. If there are multiple j with $C_j > 1$, $\tau - 1$ chooses one of them randomly. (This last case is off the equilibrium path.) Thus the hypothesis holds for all t. ■

Given the preceding lemmas, we will now focus on characterizing the equilibrium decisions of agents after observing history $h_t^{'}$ (i.e. with $C_j = 1, \forall j \leq t$). Before the characterization of an equilibrium string, we give some examples for purposes of exposition. Let the total number of agents be N=6. Throughout we follow a common tie-breaking rule: If an agent is indifferent between I and NI, he chooses to investigate.

Example 1 *The equilibrium string is [NI, NI,* I_2*,* I_1*,* I_3 *,* I_4 *] for some parameter values.*

To see whether this can be an equilibrium for some parameter values, we have to check whether all six no-deviation conditions can be satisfied simultaneously. Note that 1 gets future payoff only when 3's investigation of 2 is a failure, which has a probability of $1 - p_0$ *and 4's subsequent investigation of 1 is useful (probability* p_0*). The condition for 1 is irrelevant here since he has no choice effectively. His future payoff is always higher than the current one which is* 0.

$$0 < (1 - p_0)p_0(\delta^3 w + \delta^4 w + \delta^5 w) = (1 - p_0)p_0\delta^3 w(1 + \delta + \delta^2)$$

The condition for 2 however is that the current net payoff be lower than the expected future payoff i.e.

$$v - \frac{c}{p_0} < p_0\delta w(1 + \delta + \delta^2 + \delta^3)$$

Now if 3 deviates, no citation is observed for 2. Hence no future player would investigate 2, but 4 investigates 1. 3 would be investigated by 5 only if 4's investigation is not useful(probability $1 - p_0$). So, 3's expected future payoff from deviating is $(1 - p_0)p_0\delta^2 w(1 + \delta) = Q$(say) while his payoff from investigating is $p_0 v + (1 - p_0)Q - c$. the condition needed for 3 not to deviate from I_2 is

$$v - \frac{c}{p_0} \geq (1 - p_0)p_0\delta^2 w(1 + \delta)$$

For 4, the condition is

$$v - \frac{c}{p_0} \geq (1 - p_0)p_0\delta^2 w$$

while for 5 and 6 it is simply $v - \frac{c}{p_0} > 0$.

So, the parameter values needed to sustain the specified equilibrium string should satisfy

$$L' = (1 - p_0)p_0\delta^2 w(1 + \delta) \leq v - \frac{c}{p_0} < (1 - p_0)p_0\delta^3 w(1 + \delta + \delta^2) = H' \quad (1)$$

We see this is possible for δ high enough.

Example 2 *Now let us consider the equilibrium string [NI, I_1, NI, I_3, I_4, I_5]. We will check if there exists some values of parameters such that no one deviates from this equilibrium.*

Conditions needed for this to be an equilibrium are:
1: $0 < p_0\delta w(1 + \delta + \delta^2 + \delta^3 + \delta^4)$
2: $v - \frac{c}{p_0} \geq 0$
3: $v - \frac{c}{p_0} < p_0\delta w(1 + \delta + \delta^2)$
4: $v - \frac{c}{p_0} \geq p_0\delta w(1 + \delta)$
5: $v - \frac{c}{p_0} \geq p_0\delta w$
6: $v - \frac{c}{p_0} \geq 0$
Hence the condition needed is

$$L = p_0\delta w(1 + \delta) \leq v - \frac{c}{p_0} < p_0\delta w(1 + \delta + \delta^2) = H \quad (2)$$

So we see from these two examples that both the strings can be equilibrium strings depending on whether the values of parameters satisfy the respective conditions. Now we can in fact show that the two ranges of $v - \frac{c}{p_0}$: [L,H] and [L',H'] may not be disjoint. If they are not, then given that parameters satisfy (1), we cannot be sure that the string is as in Example 1. So, for some parameter ranges, both (1) and (2) might be satisfied and hence there can be multiple equilibria.

Hence we see that there is an irreducible multiplicity of equilibria, pure as well as mixed and we cannot make any precise predictions regarding the pattern of investigations by agents. Note that, crucial to this multiplicity is the behaviour of agents when indifferent between investigating two or more agents. However, if we impose some rules (corresponding perhaps to social norms in the fields concerned-see the next subsection) on how agents behave if they are indifferent, we can obtain partial characterisations of equilibrium behaviour. We now turn to these.

3.1 Behavioural Assumptions

In case some entrant is indifferent investigating among a set of agents, he can choose any one or mix between them. We impose some simple behavioural rules in these cases. Suppose entrant k is indifferent between agents $1, 2, ...k-3$. Some of the simpler rules could be (1) k chooses the earliest i.e. 1 to investigate or (2) k chooses the most recent agent i.e. $k-3$ or (3) he mixes between all of them with equal probability. Hence we focus on two types of pure strategies and one completely mixed strategy. These could be thought of as extreme cases of some regularities observed in practice. Price [20] observed that different subjects can be categorised into two classes: classical or ephemeral. Subjects like Physics and Engineering are *ephemeral* i.e. recent papers tend to be cited more often while Geology, Mathematics are *classical* ; they cite more of the older papers. Some subjects, however, show no clear trend. We take our cue from these observations and analyse the game with these three behavioural assumptions.

Any new entrant:

BA1: If indifferent among r agents, investigates the earliest among them.

BA2: If indifferent among r agents, investigates the most recent agent.

BA3: If indifferent, mixes among all r agents with equal probability.

Now, we focus on each of these at a time and characterise the equilibrium string corresponding to each. The proofs are relegated to the Appendix.

Behavioural Assumption 1: *Any new entrant, if indifferent between r agents, investigates the earliest among them.*

Proposition 1 *In any equilibrium string, for $N > 4$, $\exists K^* \leq \frac{N-4}{2}$ s.t.$\forall k \leq K^*$,the k^{th} entry is NI and $\forall k > K^*$, the k^{th} entry is I. The exact value of K^* depends on the parameter values v, δ, p_0, w.*

Proof. *The proof involves 2 steps. Step 1 shows that number of agents choosing NI is less than that choosing I, in equilibrium. This is because we consider pure strategies i.e. k agents investigating implies exactly k agents are investigated. So, no more than k agents would like to choose NI in order to be investigated. In fact, it can be shown that number of agents choosing NI is less than $\frac{N-4}{2}$, given our assumptions on parameters.*

The next step involves showing that there will be no gaps. Suppose an agent i finds it profitable to choose NI and let his payoff from doing so be U_i. Then the agent j preceding him must also find it profitable to choose NI since $U_j > U_i$. This increase is due to two things: one, i will be investigated later than j conditional on j being not useful and hence the unconditional payoff is lower; second, conditional on being useful, j would get one extra citation than i would potentially get on account of being an earlier entrant. Hence j would also choose NI and so would any agent entering before i.(See Appendix 1 for details) ■

Remark 1 *This implies that there will be no gaps in equilibrium. That is, if, say, agent 5 is the first one to investigate and his investigation is not useful so that agent 6 observes no citations, then in equilibrium, it cannot be that agent 6 chooses NI and writes on his own. K^* is the entrant who first starts investigating in equilibrium. i.e. the first player for whom the expected current benefit outweighs the prospect of future benefits from being cited. This agent is ready to forgo possible payoffs of w from each future citer to get the current benefit v. Players before him, i.e. those who choose to write on their own, do not want to investigate some earlier agent and cite him, since in that case, they would never be cited.*

Remark 2 *Also, $K^* \leq \frac{N-4}{2}$ implies that the total number of agents not investigating is strictly less than those investigating, in equilibrium. The number of citations though would depend on the outcome of those investigations and is bounded above by the number of agents investigating. The exact value of agent K^* depends on the parameter values. For given w, p_0, δ the higher the v the higher the incentive for earlier players to Investigate and not wait, since the future payoff relative to v is not high enough. So, higher the v, the lower the K^* i.e. investigations start earlier.*

Behavioural Assumption 2: *Any new entrant, if indifferent between r agents, investigates the most recent among them.*

Proposition 2 *In any equilibrium string, $\exists \overline{K}, s.t. \forall i \leq \overline{K}, \forall j \leq \overline{K} - 1$, i: $NI \Rightarrow i+1 : I$ and $j : I \Rightarrow j+1 : NI$ and $\forall i > \overline{K}$, $i : I$ .The value of $\overline{K}$ depends on parameter values and, for fixed w, p_0, δ, is decreasing in v.*

Proof. *First note that two consecutive NI is not an equilibrium since the earlier agent will not be investigated and hence would deviate to I. So, a NI must be followed by a I. Also, two* *consecutive Is preceded and followed by NIs cannot be an equilibrium either. Suppose $i, i+1$ chooses I and agents $i-1$ and $i+2$ choose NI.In this case, agent i finds it profitable to investigate which implies that $U_i(I) > U_i(NI)$. Now for agent $i+2$, $U_{i+2}(NI) < U_i(NI)$ since there are less number of agents entering after him.Since utility from investigating is same for all player and equal to $v - \frac{c}{p_0}$, given agent i is choosing optimally, agent $i+2$ should deviate since $U_{i+2}(NI) < U_i(NI) < U_i(I)$. Ruling out these patterns leaves the specified pattern as an equilibrium depending on parameter values. (Appendix 1 for details)* ■

Behavioural Assumption 3: *Any new entrant, if indifferent between r agents, investigates them with equal probability $\frac{1}{r}$.*

Proposition 3 *In any equilibrium string, $\exists \widetilde{K} \leq 2, s.t. \forall k < \widetilde{K}$ the k^{th} entry is NI and $\forall k \geq \widetilde{K}$, the k^{th} entry is I . In fact, $\widetilde{K} = 2$.*

Proof. Note that if $i, i+1$ chooses I and $i-1$ chooses NI, then only i mixes (by Lemma 1).

Let the $i_1^{th}, i_2^{th}, ...i_k^{th}, i_{k+1}^{th}, i_{k+2}^{th}, ...N^{th}$ agents be the ones choosing I in equilibrium; $i_1 < i_2 < .. < i_k$.Hence i_k is the agent after which there is no agent choosing NI in equilibrium and i_1 is the first agent to choose I. From the conditions for no unilateral deviation by agents i_k and $i_k - 1$, the parameters should satisfy the following condition:

$$p_0 w\delta(1 + \delta + ... + \delta^{N-i_k-1}) \leq v - \frac{c}{p_0} < \frac{1}{i_k - i_{k-1}} p_0 w\delta(1 + \delta + ...\delta^{N-i_k})$$

the necessary condition for this to hold is

$$or, \ (1 + \delta + ... + \delta^{N-i_k-1})(i_k - i_{k-1}) < (1 + \delta + ...\delta^{N-i_k})$$

which can hold only if $i_k - i_{k-1} = 1$. Hence everyone after i_{k-1} investigates. We can redefine i_k now and do the same exercise which implies that there can be no gaps in equilibrium.

Next we can show that the number of NIs, in fact, cannot be more than 2. (See Appendix 1 for details). ■

Remark 3 *In this case we see that investigations start very early, as implied by the value of $\widetilde{K}$. Whatever be the value of v relative to the other parameters, only the first entrant waits(since he has no choice) and investigation starts from the second agent. This equilibrium entails the maximum number of investigations and expected citations.The intuition behind the early investigations is that by the strategy of randomising among agents when indifferent,* *each early entrant, say t faces elimination from the race even when some other agent is investigated. If the investigated agent's idea was useful, then t will obviously get no future citations. But even when the idea is not useful, the new entrant would not investigate t since the probability that t was investigated is positive. This significantly reduces incentives to write on one's own and try to get future citations-payoffs resulting in investigation starting early in the process.*

Now, we can compare these different equilibria in terms of when investigations start. BA3 obviously involves early investigations. We can compare K^* and $\overline{K}$ for given parameter values and this will tell us which type of behaviour entails early investigations and hence early revelation of a high-quality (always useful) paper . We can show that for any set of parameter values, BA1 induces earlier revelation of the quality of papers compared to BA2. i.e. $\widetilde{K} \leq K^* < \overline{K}$ for any given parameters.

Proposition 4 *For any given set of parameters, $(\upsilon, p_0, w, \delta)$, $\widetilde{K} \leq K^* < \overline{K}$.*

Proof. To show this we need to fix K^* at a particular value, say Y. This corresponds to some range of parameter values, set υ, say. Now each $\overline{K}$ corresponds to some set of parameter values, say ω. For a $\overline{K} \leq K^*$, call the set ω_l. We can show that ω_l does not intersect υ. Hence, given $K^* = Y$ i.e. given that parameters lie in υ, they cannot belong to ω_l which implies that $\overline{K} \nleq K^*$.(See Appendix 1 for details). ■

Following the characteristics of the equilibria outlined in this section, we plot the expected discounted number of citations for each entrant under the different behavioural assumptions. The parameter values for Fig 3 are $\delta = 0.99, N = 20, p_0 = 0.5$ and υ, w, c such that $K^* = 4, \overline{K} = 7$.

3.2 The infinite horizon model

In the previous section we considered a finite number of agents making decisions of investigating and citing. One natural question that arises is what happens when there is no known bound on the number of agents. In this section, we modify the basic model by considering an infinite horizon game where one agent enters in each period. The rest of the game is as before. After entry an agent observes the state of citations for the existing agents and updates his priors regarding the usefulness of a paper. Then he decides whether or not to investigate one of the existing papers. We will focus on the stationary equilibria of this game where the strategy depends only on the citations observed.

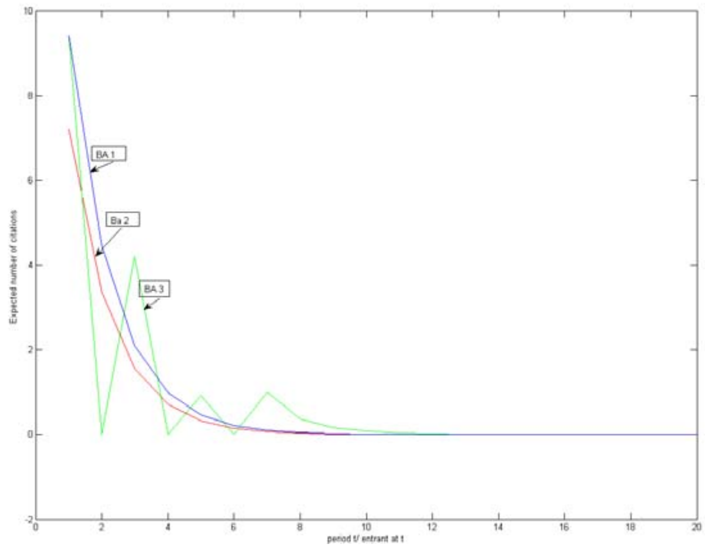

Figure 3: Expected number of citations

To characterize the equilibrium stationary strategy, one needs to define the history at any time t, h_t. At any time t, there could be two types of histories: (i) $h_t^1 = h_t(C_\tau > 1$, for some $\tau < t)$ or (ii) $h_t^2 = h_t(C_\tau = 1, \forall \tau < t)$, i.e. a history with one or more citations or one with no citations (apart from self-citations). The stationary strategy of agent t can be denoted by $s_t = \{s_t^1, s_t^2\} = \{s(h_t^1), s(h_t^2)\}$. The next proposition characterizes the equilibrium stationary strategy, s^*.

Proposition 5 *The unique stationary equilibrium is (i) $s^* = \{I, I\}$ for all t if $\frac{p_0 \delta w}{1-\delta} < v - \frac{c}{p_0}$ and (ii) $s^* = \{I, \sigma(\lambda)\}$ if $\frac{p_0 \delta w}{1-\delta} > v - \frac{c}{p_0}$ where $\sigma(\lambda)$ denotes the mixed strategy with λ being the probability of investigating.*

Proof. The proof proceeds by first characterizing the equilibrium strategy for $h_t = h_t^1$. From the previous analysis it is obvious that $s^*(h_t^1) = I$. Next, note that $s^*(h_t^2) \neq NI$. Suppose it is. Therefore, when agent t observes no citation, then he chooses NI. This in turn implies that $h_{t+1} = h_{t+1}^2$ and since $s^*(h^2) = NI$, agent $t+1$ chooses NI. Hence, agent t has no current or future payoff and is better off deviating to investigating some $\tau < t$ and getting an expected payoff of at least $p_0 v - c > 0$.

The next claim is that $s^*(h^2) = I$ is an equilibrium for some parameter values. Note that the given equilibrium strategy implies that agent 2 investigates 1, 3 investigates 2 at history

h^2 and so on. This also implies that at history h_t^2 agent t knows that the investigations of agents 2 through $t-1$ have been unsuccessful and hence the probability of usefulness of agents 1 through $t-2$ is zero. Hence agent t would investigate agent $t-1$. Note that this also implies that in the event that $t+1$ faces history h_{t+2}^2, he will investigate agent t. Therefore, the expected payoff of t from I can be written as

$$\begin{aligned} E\pi_t(I) &= p_0 v - c + (1-p_0)p_0(\delta w + \delta^2 w + \ldots\ldots) \\ &= p_0 v + (1-p_0)\frac{p_0 \delta w}{1-\delta} - c \end{aligned}$$

If t deviates to NI, he gets

$$\begin{aligned} & p_0(\delta w + \delta^2 w + \ldots\ldots) \\ = & \frac{p_0 \delta w}{1-\delta} \end{aligned}$$

So, if $v - \frac{c}{p_0} > \frac{p_0 \delta w}{1-\delta}$, then agent t chooses I. Hence for this set of parameter values the unique stationary equilibrium is to investigate the immediately preceding entrant with probability 1.

If $v - \frac{c}{p_0} \leq \frac{p_0 \delta w}{1-\delta}$, then the pure strategy $s^* = \{I, I\}$ is not an equilibrium. Let the mixed strategy σ be the following: When $h_t = h_t^2$, investigate the previous entrant with probability λ and choose NI with probability $1-\lambda$. For σ to be an equilibrium it must be the case that

$$\begin{aligned} E\pi_t(I) &= E\pi_t(NI) \\ & \quad or \\ p_0 v - c + (1-p_0)\lambda p_0 w(\delta + \delta^2 + \ldots.) &= \lambda p_0 w(\delta + \delta^2 + \ldots) \end{aligned}$$

This holds for $\lambda = \frac{1-\delta}{p_0 w \delta}[v - \frac{c}{p_0}] \in (0,1)$ for $v - \frac{c}{p_0} \leq \frac{p_0 \delta w}{1-\delta}$. ■

Thus, each agent would choose to investigate with some probability λ until the first time the investigation is successful, after which everyone would cite the successful paper. Thus there would be a probabilistic "monopoly"with the ex ante probability that the t^{th} entrant is the monopolist being $(1-\lambda p_0)^{t-2}\lambda p_o$.

4 Private Information

In this section we consider the case where each player receives a private signal about the 'quality' of his paper before taking any decision[11]. We represent this signal as an agent-specific probability of success and denote it by p_i for the i^{th} agent with $E(p_i) = p_0$. Let

[11] Baliga and Sjostrom ([2]) design a mechanism for self-assessment and peer review (in a different context) based on a similar assumption.

the distribution of p_i be denoted by $F(.)$ and let p_i be i.i.d across agents. Now, consider the basic model with one entrant each period; if useful a paper yields the same value v. The cost of investigation is c, as before. Let r_{t+1} be the probability of agent $i+1$ (entering at $t+1$) investigating any of his predecessor(which can be anything $\in [0,1]$). The i^{th} agent's decision between investigating and not at any time will depend on his type.

At any time t, he can choose one of the two and get the corresponding payoff.

Investigate: $p_0 v + (1-p_0)\delta p_i r_{t+1} W_{t+1} - c$ where $W_{t+1} = (w + \delta w + ... + \delta^{N-i-1} w)$, which represents future payoffs from being cited.

Not Investigate: $\delta p_i r_{t+1} W_{t+1}$

Therefore *NI* is chosen when

$$p_0 v + (1-p_0)\delta p_i r_{t+1} W_{t+1} - c < \delta p_i r_{t+1} W_{t+1}$$

$$or,\ v - \frac{c}{p_0} < \delta p_i r_{t+1} W_{t+1}$$

$$or,\ p_i > \frac{v - \frac{c}{p_0}}{\delta r_{t+1} W_{t+1}} = p_t^*. \tag{10}$$

Then the probability that i investigates a predecessor is $F(p_t^*) = r_t$.Hence for every period there is some cutoff type p_t^* such that all $p_i < p_t^*$ entering at time t will investigate some predecessor.

Let the cutoff levels of the types for each period be represented by $p^* = (p_1^*, p_2^*, ...p_{N-1}^*, p_N^*)$. This sequence also defines the sequence of $r^{*\prime}s$ by the relation $F(p_t^*) = r_t\ \forall t$.If there are N time periods (or equivalently, N entrants), $r_N = 1$. Likewise, depending on the values of v, w, p_o, δ, c, all entrants from some $k+1 \leq N$ onwards will investigate with probability 1, and k is the last time period for which $r_k < 1$. The rest of the sequence is defined recursively by $\frac{v-\frac{c}{p_0}}{\delta F(p_{t+1}^*) W_{t+1}} = p_t^*$.

Now given this sequence we can argue that in equilibrium, if an agent i investigates a predecessor, it has to be $i-1$ whom he investigates. This is a direct consequence of Lemma 1. Conditional on observing no citations, the probability that $i-1$ is of type p_i is $F(p_i)$, so that ex-ante probability of $i-1$ being useful is $E(p_i) = p_0$. Some agent $j < i-1$, on the other hand has been investigated by some agent $p_k < p_j^*$ which occurs with positive probability. Hence by Lemma 1, the probability of agent j being useful $< p_0$. So, if i investigates at all, he will investigate $i-1$. This gives some some justification for the behavioural assumption 2 that we imposed in the main analysis. (Unfortunately, this was the least efficient one.)

One odd feature of this setup is that the sequence of cutoffs alternates in size-a high probability of citing next period leads to a lower probability today, other things being equal, and vice versa.

5 Private Information with Strategic Entry

In this section we further extend the model to include entry as a strategic decision. If agents know their type and are free to choose when to enter, then we might observe some sorting regarding timing of entry. Each agent receives a signal about his type p_i before the start of the game. He, agent i, has two decisions to make at every time period t; to 'enter , E' or 'Wait one period, W'. If he chooses E, he then chooses to I(nvestigate) or NI, as before. If he chooses W, then at time $t+1$, he again has the same choices. All agents make these choices simultaneously. So, the decisions are functions of their types (and of course, the history at any time t) only.

Let the distribution of p_i be i.i.d uniform [0,1].[12]Let T be the fixed number of periods and $T > 2$.

Proposition 6 *Let N, w, v be such that $(N-1)w > v$.Then there exists an equilibrium described by the cutoffs α_j, $1 > \alpha_1 > \alpha_2 > ...\alpha_{T-1} > 0$ with $\alpha_t = \alpha_1^t$ such that:*

1. *If $p_i \geq \alpha_1$, Player i enters in period 1. If there is at least one entrant in period 1, all other players enter in period 2.*

2. *If there are no entrants for periods t=1,2,..$\tau < T-1$, Player i enters if $p_i \geq \alpha_{\tau+1}$.If there is at least one entrant in any period τ, all other players enter in the following period.*

3. *The players who (simultaneously) enter first do not investigate. Those who enter in the following period investigate.*

4. *If the first entry occurs at period τ, any player who has not entered by period $\tau+1$ does so in $\tau+2$.*

5. *In period T, everyone who has not entered, enters.*

Proof. Note first that the histories in this game are characterised by the identities of the players who enter in each period. The *state* of the game is given by $(a_t, k_\tau, \tau \leq t)$ where the distribution of p_i after period t is uniform $[0, a_t]$ and k_τ is the number who have entered at period τ. We shall limit ourselves to strategies that depend only on the state and not on the identities of the players who have entered at different periods.The only out-of-equilibrium moves we need to consider are given by point 4 above. The effect of such moves on beliefs is irrelevant for the equilibrium strategies.

[12] This is without much loss of generality and saves on notation.

Suppose no player has entered by period τ. Then the belief $a_\tau = \alpha_\tau$. The probability that any player will enter is then $\gamma_{\tau+1} = \frac{\alpha_\tau - \alpha_{\tau+1}}{\alpha_\tau}$.[13] Let $m_{\tau+1} := 1 - \gamma_{\tau+1}$. Also, let $\widehat{p}_{\tau+1}$ be the highest expected probability (entrants at different times might have different probabilities of being useful) at time $\tau+1$ that any of the earlier entrants is useful. Consider Player i in period τ, conditional on no previous entry. If he enters and k others out of $N-1$ enter then his expected payoff conditional on k is given by:

$$p_i \delta \frac{N-k-1}{k+1} w. \tag{11}$$

Here we are assuming each of the $k+1$ initial entrants is investigated by the others in the following period with equal probability (a version of BA3 but possibly the only reasonable assumption here). If Player i is found useful by anyone of these others who investigates him, he will be found useful by all the others who also choose him to investigate. This expression (11) is decreasing in k.

The unconditional expected payoff is therefore:

$$p_i \delta w E_k\left(\frac{N-k-1}{k+1} \mid m_{\tau+1}\right).$$

Since the term to the right of the expectation operator is decreasing in k, the expectation is decreasing in the probability of entry (by first-order stochastic dominance) and therefore increasing in $m_{\tau+1}$. If Player i chooses to wait, his expected payoff, by the equilibrium strategies, is

$$(m_{\tau+1})^{N-1} p_i \delta^2 w E_k\left(\frac{N-k-1}{k+1} \mid m_{\tau+2}\right) + (1 - m_{\tau+1}^{N-1}) \delta (\widehat{p}_{\tau+1} v - c), \tag{12}$$

These last two expressions are equal for $p_i = \alpha_{\tau+1}$.Also, by Bayes' Theorem, $m_{t+1} = \frac{\alpha_{t+1}}{\alpha_t}$. The conditional probability $\widehat{p}_{\tau+1} = \frac{\alpha_t + \alpha_{t+1}}{2}$. The cutoff $\alpha_{\tau+1}$ is defined by the following equality:

$$\alpha_{\tau+1} \delta w E_k\left(\frac{N-k-1}{k+1} \mid m_{\tau+1}\right) = (m_{\tau+1})^{N-1} \alpha_{\tau+1} \delta^2 w E_k\left(\frac{N-k-1}{k+1} \mid m_{\tau+2}\right) + (1 - m_{\tau+1}^{N-1}) \delta (\widehat{p}_{\tau+1} v - c)$$

If $\tau = T-1$, i.e. the current period is the last, $m_T = 0$. (All remaining players enter and everyone gets 0.) Suppose m_t is defined for $t = T, T-1, ...\tau+2$ and suppose the belief in period $\tau+1$ is that p_i is uniformly distributed between [0,a_τ]. We now show there exists a $\alpha_{\tau+1} \epsilon (0,1)$ such that a player i will enter if and only if $p_i \epsilon [\alpha_{\tau+1}, a_\tau]$.

[13] If $\tau = 0$, i.e. we are referring to the first period, we adopt the convention that $\alpha_0 = 1$.

Suppose, in the above equation, $\alpha_{\tau+1} = 0$. Then $m_{\tau+1} = \frac{0}{a_\tau} = 0$. Also, $\widehat{p}_{\tau+1} = \frac{a_\tau}{2}$. The left-hand side of the expression above is 0 and the right-hand side is positive, so the LHS<RHS. Now put $\alpha_{\tau+1} = a_\tau$. Now the LHS is greater than the RHS, ($\delta < 1$ even if $m_{\tau+2} = 1$ and stochastic dominance give us this inequality). But both the LHS and the RHS are continuous in $\alpha_{\tau+1}$. Therefore the LHS=the RHS for some interior value of $\alpha_{\tau+1}$.It is clear that if $p_i = \alpha_{\tau+1}$ is indifferent between entering and waiting, every $p_i > \alpha_{\tau+1}$ will enter

Therefore, in equilibrium, $a_t = \alpha_t$ for all t.[14] ■

Example 3 *Let the distribution of p_i be i.i.d uniform [0,1]. Let T be infinite.Also, let N, w, v be such that $(N-1)w > v$, and let $c = 0$.Then there exists an equilibrium described by the cutoffs α_j, 1>$\alpha_1 > \alpha_2 > ...\alpha_{T-1} > 0$ with $\alpha_t = \alpha_1^t$, which satisfies the conditions of the previous proposition.*[15]

Proof. This proceeds in the same way as the proof of the proposition except we are able to show that m_t is a constant if $c = 0$. ■

Now, we argue that non-monotonic equilibria are not possible.

Proposition 7 *There cannot be an equilibrium in which there exist α and $\alpha', \alpha < \alpha'$(say), such that all players with $p_i > \alpha'$ and some with $p_i < \alpha$ enter in period* 1, *while players with $\alpha \leq p_i \leq \alpha'$ enter in period 2 with other players entering after period 2.*[16]

Proof. See Appendix 2. ■

Note that we have discussed $t = 1, 2$ and $t > 2$. There could be equilibria in which no one moves before some $\tau > 1$. This could be sustained by a belief that anyone who enters before period τ has $p_i = 0$ with probability 1. This seems an unreasonable belief in that earlier entrants should have higher probabilities of usefulness, since by being early entrants they are giving up the advantages of using other people's work. We assume therefore that players who enter earlier than τ have probability 1 of being useful. This destroys equilibria with delay driven by beliefs.

All this is still not sufficient to claim uniqueness because the proof of the existence of the α_t did not claim the sequence was unique.

[14] We are not claiming the sequence α_t is unique.

[15] That is, once someone enters in a period, everyone else enters in the next period. The infinite horizon is needed to keep m_t constant.

[16] Using period 1 in the statement is without loss of generality-we can replace it by "period τ such that there has been no entry up to $\tau - 1$."

This result is similar to some results in the endogenous timing literature. Brian Rogers **??** and Jianbo Zhang **??** also find that in an environment with private signals about a state of the world, when actions and timing of actions of agents are made endogenous, the agent with the most precise signals acts first and all other agents mimic his actions immediately. Thus there is an information cascade (with possible initial delay). The information structure of their models differs from ours. Apart from there being no imperfect monitoring in their model, the main difference is that there is no competition, in the papers of Jianbo Zhang and Brian Rogers, among agents entering at the same time . However, even with different setups and information structures, the results have a qualitative resemblance.

6 Citations in a Random Social Network

In this section, we consider how the citation network interacts with the structure of social acquaintance. One obvious way in which social connections influence citations is that it is easier to learn of the existence of a paper through one's colleagues and friends. This might account for the frequency with which some colleagues cite each other, though there might be other issues involved there as well.

Whilst this is certainly less important now than it was in the past because of the easy accessibility of new work on the internet, it still plays a major role in pointing us to papers that reduce our search costs. Of course, another way would be to consider people who write in a given field and check whether a particular person (a "star") has worked on the specific topic, even if he or she is not an acquaintance. We shall briefly consider a "star" network later.

Here the graph of social links is assumed to be random. The agent has an existing social network in which each of the $N-1$ possible links is open with probability q and edges are open and closed independently of each other. The probability that the agent is completely isolated is therefore $(1-q)^{N-1} = 1-\rho$, say.[17]. The process then continues as follows:

1. Each period, one randomly chosen agent enters. (That is, any given agent has a probability $\frac{1}{N}$ of entering at any position in the order.)
2. Agent k upon entering realises his type p_k which is the probability that k is useful. One of the social links to the $N-1$ other agents is then activated randomly. Since each link is just as likely to be activated and just as likely to be open as any other, the probability that any given other agent will be chosen is $\frac{1}{N-1}$.

[17] Recall that in Price's work, 1% of the entering agents were completely isolated.

3. Agent k observes the agent he is linked to has or has not entered; if agent k is linked to agent j and agent j has cited some j', then k observes this and any citations j might have received.

4. Agent k then decides whether or not to investigate j or j', just as in the basic model, if j has entered.

5. If investigation of j is successful, a citation for j results. Agents get their payoffs just as in the basic model and the game reaches the next period.

There are therefore four possible states of information for k. He is completely isolated (h_0), linked to j who has not entered (h_{ne}), linked to j who has entered and has not cited anyone (h_j) or has cited j' ($h_{jj'}$). The number of citations that j has received is also observable and is denoted by C_j. A strategy for k would specify whether to investigate and whether to cite if found useful for each history. Note that k can infer something about his relative position in the order of entry based on the state of information for the last three states (h_{ne} shifts the probability attached to possible entry times towards earlier periods and h_j and $h_{jj'}$ with $C_j > 1$ towards later ones). The later the entry the lower the possible future benefits from being cited, so the incentive to hold out is lowest in the last two types of states.

Since we have two kinds of networks here, let us denote the two by g and g^c. The social network is denoted by g and the directed graph of citations is g^c. Note that g is formed randomly while g^c is formed by strategic decisions, which depend on the subgraph of g at each period. We say $ij \in g$, if they are linked socially and $ij \in g^c$ if i has cited j.[18]

Now we analyze $i's$ decision to investigate or not.

Suppose agent k enters at period k and the network at that time is given by the pair (g, g_c^k).Suppose, the number of agents who have entered is $k-1$. Agent $i's$ type is given by p_i which is distributed with cdf $F(.)$ with $E(p_i) = p_{0.}$[19] We consider the different cases that might arise. Let $W^k(C_j)$ denote the expected future payoff of the $k^{th} - period$ entrant, as calculated by the entrant (conditional on his being useful for sure) when his (social network) neighbour j has C_j citations,

Case I (occurs with probability $1-\rho$): i remains isolated. In this case, i has no option but to write on his own.

Case II: Agent k is connected to j, s.t. $C_j = 1$ and $\nexists \ell, s.t. j\ell \in g_c^t$ (i.e. j has not cited anyone). Here, k has two options:

Investigate j and get $p_0 v - c + (1-p_0) p_i W_0^k$

Not Investigate and get $p_i W_0^k$ where $W_0^k = W^k(C_j = 1)$

[18] Note that g^c is a directed graph and hence, $ij \in g^c$ is not the same as $ji \in g^c$.

[19] The distribution is absolutely continuous.

So, i will investigate if $\quad p_0 v - c + (1-p_0)p_i W_0^k \geq p_i W_0^k \quad or,\ p_i \leq \frac{v-\frac{c}{p_0}}{W_0^k}$

The probability that the new entrant investigates is denoted by $r = F(\frac{v-\frac{c}{p_0}}{W_0^k})$

So, the probability that j (with $C_j = 1$) gets a citation is $\quad \frac{\rho}{N-1}F(\frac{v-\frac{c}{p_0}}{W_0^k})p_j$

Case III: Agent i connects to j with $C_j > 1$ and/or $\exists \ell$ s.t. $j\ell \in g_c^t$.

First note that, if $\exists \ell$ s.t. $j\ell \in g_c^t$, then ℓ must be the earlier entrant. So, the first agent $\widehat{j}$ connecting to j observes ℓ with $C_\ell > 1$ and $C_j = 1$. Agent $\widehat{j}'s$ decision is to

i) Investigate $j : p_0 v - c + (1-p_0)p_{\widehat{j}} W_0^\tau$

ii) Investigate $\ell : v - c$

iii) Not Investigate: $p_{\widehat{j}} W_0^{\widehat{j}}$

So, if $\widehat{j}$ does investigate, he will investigate agent ℓ and not j.(To see this, note that if $p_0 v - c + (1-p_0)p_{\widehat{j}} W_0^{\widehat{j}} \geq p_{\widehat{j}} W_0^{\widehat{j}}$, i.e.$v - \frac{c}{p_0} > p_{\widehat{j}} W_0^{\widehat{j}}$, then it is true that $v - c > v - \frac{c}{p_0} > p_{\widehat{j}} W_0^{\widehat{j}}$). This implies that $C_j > 1$ and $\exists k$ s.t. $j\ell \in g_c^t$ cannot hold together in equilibrium. Either $C_j > 1$ **or** $\exists \ell$ s.t. $j\ell \in g_c^t$ but not both. If $j\ell \in g_c^t$ for some ℓ, then the probability of j getting a citation $= 0$.[20] We look at the case where $C_j > 1$. In such a situation, the new entrant i has two options again with the payoffs as follows:.

Investigate j: $v - c$

Not Investigate : $p_i W^k(C_j > 1)$

So, the new entrant will investigate if $p_i \leq \frac{v-c}{W^k(C_j>1)}$, which implies that the probability of getting another citation for j, given $C_j > 1$ is

$$\Pr(Citation_j \mid k) = \frac{\rho}{N-1}F(\frac{v-c}{W^k(C_j > 1)}) \tag{A}$$

Note that W^k is k's future payoff if he does not investigate any agent. Agent k cannot observe the whole graph (g, g_c) but only the one he is connected to and anyone this person has cited. So, k has some expectation of the period of his entry, which determines W^k. Observation of a higher C_j implies that more people have entered and hence k has entered relatively late. This in turn implies that the W^k is low. Note that Player $k+\tau$, $\tau \geq 1$,if she links to k and no one else has cited k, will be in information state h_j and will not assign a high probability to being late in the game. Therefore $k+\tau$ will cite with a relatively low probability independent of the value of C_j. Therefore C_j does not affect the probability of citation for k but does affect the expected number of citations, conditional on being cited.

[20] Of course this could occur off the equilibrium path. In this case, the new entrant believes that all those who have cited J made mistakes and cites the person j has cited. (This is an assumption on beliefs but a natural one.)

So, a higher C_j implies a lower $W^k(C_j)$.[21] From expression A, we see that this implies a higher $r = F(\frac{v-c}{W^k(C_j)})$ and a higher $\Pr(\text{Citation}_j)$.

However, this is the probability of citation if k links directly with j. If k links to someone who has linked to j, the probability k cites j is independent of C_j and is lower. Note therefore that the probability of j being cited with $C_j > 1$ earlier citations is proportional to $C_j F(.) < C_j$. We write this as a proposition.

Proposition 8 *Suppose $C_j(t)$ denotes the number of citations j has received at time t and $C_j(t) > 1$. Then*

$$\begin{aligned} C_j(t+1) &= C_j(t) + 1 \text{ with probability } \varphi(C_j, t) \\ &= C_j(t) \text{ otherwise.} \end{aligned}$$

Here

$$\varphi(C_j, t) = \frac{\rho}{N-1}[F(\frac{v-c}{W^t(C_j > 1)}) + (C_j - 1)F(\frac{v-c}{W_0^t})]$$

Proof. See preceding discussion. ■

Remark 4 *The probability of an additional citation is therefore increasing in the number of citations as in the preferential attachment models, but unlike these models is not directly proportional to the number of existing citations. The probability has two factors, one arising from the social network and the other from the strategic/competitive motives of the players. The sublinear nature of the dynamic does not give a power law (as Chung et al have shown).*

We have earlier referred to a star network. Suppose the network consists of "stars", who might be connected to each other, and "planets", who revolve around particular stars. In this case, if a star enters early, his paper will be likely to receive wide dissemination. However, an idea generated from a planet can only diffuse if a star decides to cite it. This can only happen if a star enters relatively late, so has no incentive to seek her own citations. Thus ideas generated from the peripheries take an inefficiently long period of time to diffuse.

We now give an example where the strategic aspect of network formation is absent, so as to give a flavour of the effect of the acquaintance network on citations.

Example 4 *Let N=10. Say, 1 and 2 entered sequentially but no social link was formed. So, $C_1 = C_2 = 1$ at period 3. 3 enters and forms a link with 2, say. Suppose 3 chooses NI. So, after period 3 the network is: $<1(1) \quad 2(1)–3(1)>$*

[21] Player i has a link with a player who has cited Player j who has C_j citations. Therefore Player i has been preceded by at least $C_j + 1$ agents.

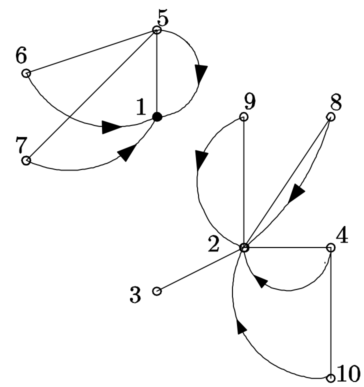

Figure 4: Co-evolution of citation and social networks

The numbers in the parentheses are the citations for each agent. Now 4 enters. Suppose he links to 2 and investigates 2. Also suppose it is useful. Then the updated network is <1(1) 2(2)—3(1) 4(1) >.Now say 5 links to 1 and investigates and cites 1. 6 and 7 link to 5 and he can observe $C_1 > 1$ *and hence cites 1 again. Hence at the end of period 7, 1 has 3 citations and is linked to 5,6,7 in* g *or* g^c*. Say, 8,9, and 10 all link to 2 or 4. In either case they can observe* $C_2 > 1$ *and hence would cite 2. So, the final graph* $< g^c(g) >$ *we end up with looks like figure 4. We could have the same* g^c *with multiple* g*'s. Figure 4 includes only one of the possible* g *which is consistent with the* g^c*.*

7 Private information and heterogeneous quality

In this subsection we discuss heterogeneity in the qualities of papers with private information. We explore whether greater heterogeneity in the quality of papers will speed up or delay the revelation of information about quality. An individual's paper might be of quality 0 (not useful for related papers) with probability $1 - p_i$;useful with a value $\underline{v}$, with probability $p_i(1 - q)$, or with value $\overline{v}$, with probability $p_i q$.We still maintain that $h = 1, l = 0$ i.e. once useful(non-useful), a paper is always useful(non-useful).The quantity p_i is, as in the preceding section, private information for Player i and is drawn independently for each i from a commonly known, absolutely continuous distribution on [0,1], with $E(p_i) = p_0$. We assume

i) $\underline{v} > w$

ii)$v = q\overline{v} + (1 - q)\underline{v}$

$iii) p_0 v > c$ (from the basic model)

Therefore the prior probability of a paper yielding $\overline{v}$ is $p_0 q$ and that of it yielding $\underline{v}$ is $p_0(1-q)$; while the probability of a paper not being useful or of value 0 is $(1-p_0)$ as before. The difference comes from the fact that if both $\overline{v}$ and $\underline{v}$ types are cited if used, a citation does not partition the two useful types though it does separate them from a non-useful paper. So, there is partial information revelation, i.e. citations are not perfect signals any more. In this setup, an agent might choose to incur cost c and investigate a paper if the ex-ante net payoff from doing so is high enough. After reading, the information is revealed and depending on the quality, he might or might not cite that paper. We consider N entrants in a fixed order.

We now introduce some notation prior to outlining the result. Let r_k be the probability that Player k will investigate one of his predecessors. (For $k = N$, the probability is 1). We consider a $k < N$, again such that there have been no citations upto $k's$ entry. If Player k does not investigate or investigates and does not cite, her expected value to others will be updated, since the players with higher values of p_k will be more likely not to investigate. (If $p_k = 1$, the player knows his paper is useful and therefore has a high expected future payoff from being cited). Suppose her probability of being investigated is r_{k+1}[22] and her expected payoff, if cited is W_{k+1}. Note that if Player k is not found useful, she is not cited, but if she is found useful, she is cited with probability r'_{k+1} if her value is $\overline{v}$ and with probability r''_{k+1} if her value is $\underline{v}$, where $r'_{k+1} \geq r''_{k+1}$.If cited once, her expected value to future entrants is at least $q\overline{v} + (1-q)\underline{v}$, which is always greater than the expected value of someone who has never been cited. However, if she is cited again, her expected future value increases and if she is not cited it decreases. This changes the investigation decision for future entrants. This is all encapsulated in W_{k+1}. Thus r_k, W_k are well-defined (by backward induction) for all k.

Proposition 9 *Suppose $(\overline{v} - \underline{v})p_0 q > c$, where c is the cost of investigation. Then, for every k, given no citations before k, there will exist cutoff values, $\alpha_1 > \alpha_2 > \alpha_3$[23], such that Player k will not cite if $p_k \geq \alpha_1$, will not investigate if $p_k \geq \alpha_2$, will investigate and cite only $\overline{v}$ if $p_k \epsilon [\alpha_3, \alpha_2]$ and will investigate and cite both $\overline{v}, \underline{v}$ if $p_k \epsilon [0, \alpha_3]$.*

Proof. Given investigation a player with private information p_k will not cite $\overline{v}$, if $p_k r_{k+1} \delta W_{k+1} \geq \overline{v}$.This gives $\alpha_1 = \frac{\overline{v}}{r_{k+1}\delta W_{k+1}}$. Clearly, someone who is not going to cite even $\overline{v}$ would never investigate. Similarly, if $p_k \epsilon [\frac{\underline{v}}{r_{k+1}\delta W_{k+1}}, \frac{\overline{v}}{r_{k+1}\delta W_{k+1}}]$, Player k would cite only $\overline{v}$.For lower values of p_k, she would cite both positive values. Let $\frac{\underline{v}}{r_{k+1}\delta W_{k+1}} = \alpha_{3.}$

[22] Once again, the probability of citation will be shown to be positive in every period, so lemma 1 will in fact ensure that only the most recent paper is investigated, in the absence of a citation.

[23] These cutoffs depend on k; this dependence is suppressed for notational convenience.

Consider now the player who would, if she investigates and finds a positive value, cite only $\overline{v}$. Her choice would be not to investigate if $p_k r_{k+1}\delta W_{k+1} \geq p_0 q\overline{v} + (1-p_0 q)p_k r_{k+1}\delta W_{k+1} - c$,or , $p_k \geq \frac{\overline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1} p_0 q} = \alpha_2 < \alpha_1$.For $\alpha_2 > \alpha_3$,$(\overline{v} - \underline{v})p_0 q > c$ must be satisfied (and conversely). We now check the investigation decision for a player who would cite both $\overline{v}$ and $\underline{v}$, if he investigates. Such a person would not investigate if $p_k \geq \frac{q\overline{v}+(1-q)\underline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1} p_0}$.

We check the difference between the right-hand side of the last expression and α_3.This difference is $\frac{q\overline{v}+(1-q)\underline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1} p_0} - \frac{\underline{v}}{r_{k+1}\delta W_{k+1}}$

$= \frac{1}{p_0 r_{k+1}\delta W_{k+1}}(p_0 q\overline{v} + p_0\underline{v} - p_0 q\underline{v} - p_0\underline{v} - c)$

$= \frac{1}{p_0 r_{k+1}\delta W_{k+1}}(p_0 q(\overline{v} - \underline{v}) - c) > 0.$

This implies that the player with $p_k \leq \alpha_3$ would always investigate, because as p_k rises, she would shift first to citing only $\overline{v}$ (after investigating) before choosing not to investigate. ■

Note that, by lemma 1, in equilibrium, Player k, if he cites, will cite $k-1$ if no previous papers have citations other than self-citations. A paper that has received a citation will be chosen by any future entrant who wishes to investigate (and the cutoffs in the proposition will change to reflect the new expected value, obtained by Bayesian updating). However, someone investigating who cites only $\overline{v}$ papers might discover the cited paper is $\underline{v}$ and not cite it. Every non-citation will decrease the expected value of the paper and it is possible this will go below the prior, in which case the most recent paper will again start to be investigated. Thus it is possible that several papers will obtain citations and then die out and be replaced by others. As a cited paper adds citations, it will, of course, become more popular. As it accumulates non-citations, the entrants who would wish to cite only $\overline{v}$ papers might switch more to not investigate, so the information content of more non-citations would diminish. This also depends of course on how close to the end of the game the field is, because every type of agent has an incentive to investigate at the end of the game. We can therefore conclude that, with heterogeneous quality, (i) a higher quality paper has a higher probability of being cited and a higher expected number of citations; (ii) with positive probability a lower quality paper will be cited first and obtain citations, while a higher quality paper from a later entrant "dies"; (iii) some papers might enjoy a vogue and then be replaced by other more recent ones.

8 Efficiency

This paper is an attempt to model observed patterns of citation as a result of strategic choice by rational agents and its implication on diffusion of knowledge. The irreducible multiplicity of equilibria makes determinate predictions difficult. But we can rank these equilibria with respect to a certain notion of ex-ante efficiency. Efficiency here relates to the idea that the earlier the investigations start, the earlier the information about the quality of paper is revealed, probabilistically. Hence potentially, the benefits of a good paper are available earlier. Consider a social planner who wants to maximise the sum of expected payoffs of all N agents. Say, the planner specifes that in states with no citations, agents 1,2,...$i-1$ would not investigate and all agents $i, i+1, ...N$ investigate. Also, whenever a paper gets a citation, it gets cited by all entrants thereafter. This allocation of decisions entails a payoff of $v+w-c$ for each citation ($v-c$ to the one citing, w to the one cited). If agent i starts investigating, agents 1,2,...$i-1$ get no benefit nor do they incur any cost. So, the sum of expected payoffs of N agents is

$$\begin{aligned} U &= \Pr(i's \text{ investigation is a success})[(v+w)(N-i-1)] \\ &\quad + \Pr(i's \text{ investigation failure}) \Pr(i+1's \text{ investigation success})[(v+w)(N-i-2)] \\ &\quad + ... \\ &\quad + \Pr(\text{investigations of } i, i+1, ..N-1 \text{ failures}) \Pr(N's \text{ investigationn success})[v+w] \\ &\quad -(N-i-1)c \end{aligned}$$

The expression is strictly decreasing in i, for small c (since the last term involves the term $+ic$). So, a social planner would set $i=2$ to maximize U. Hence we see that earlier investigations entail higher aggregate payoffs. The equilibrium in Proposition 3 , therefore, is efficient, both ex-ante and ex-post. In fact, when citations are perfect signals, as in the basic model, there is no difference between ex-ante and ex-post efficiency.

With multiple qualities of papers, the ex-ante probability that the better paper is cited and known is higher than the probability of the worse paper being known.[24] Note that existence of equilibria mentioned in section 4(with agent i choosing to cite both types and $i+1$ choosing to cite only the high type) implies that ex-post, it might be the case that the low-quality paper gets citations before a high quality one and potentially better papers do not get known. So, there are equilibria that are ex-post inefficient.

9 Conclusion

We have looked at a specific stylised model on the effect of rivalry on the diffusion of useful ideas. Whilst we have focused on academic citations, the model can be interpreted without

[24] Compare this with the David-Simkin-Roychowdhury explanation, where there is no expected difference in quality between highly cited and less cited papers, a somewhat counter-intuitive conclusion that would probably have some academic administrators worried.

too much difficulty as one of firms engaged in R&D deciding whether to use existing patents or to work around them.

Our basic findings are: (i) In a complete information model, the rule by which a new entrant chooses to cite the work of earlier entrants among whom she is indifferent determines the equilibrium. The most efficient case for dissemination of ideas is the rule by which the new entrant chooses randomly. The structure of the equilibrium often, but not always, has a cutoff entrant such that all who enter earlier decide not to investigate earlier work and those who enter later do. (ii) WIth private information and entrants deciding when to enter, the equilibrium structure is monotonic in that players who believe their own ideas to be relatively good enter early and there is then a cascade, similar to information cascades in the literature. (iii) With citations superimposed on a simple social network (so that individuals find out about other earlier work by direct acquaintance or simple word of mouth), the dynamic of citations is shown to follow sublinear preferential attachment. (iv) In no case, do we get a "power law". We get either a monopoly or, with sublinear preferential attachment, something involving the product of a power law with some other factor.

Our findings can also be related to the literature on diffusion of technology and social norms, which point out that "local externalities" like conformity can be a possible obstacle to the spread of optimal technology. Papers by Munshi [19] and Banerjee-Duflo [3] deal with specific externalities. In our model, it is the rivalry or competition among agents that becomes the hindrance to speedy diffusion. This is in contrast to existing literature where agents are usually assumed to be non-strategic. (Diffusion in the case of partially rational players who do not compete with each other is addressed in a different context by [1] and for non-rational players by [7].)

We can place our work in the network literature (citations being directed links), but the flavour is different from many papers in that literature, since we do not rely solely on exogenous randomness or on built-in network externalities. In the model without private information, our result might be considered too extreme in that there is one randomly chosen centre in a star network. In order to match the data, we need to include other factors, which contribute to the decision of citation. In our paper, private information about quality contributes substantially in matching model results with the qualitative features of the data. Additional considerations arising from repeated interactions and asymmetries in initial social connectedness among agents might induce completely different strategic considerations. For example, the presence of cliques or clusters in citation networks suggest that in a repeated game framework, (some) citations might occur in the hope of getting favours returned. In fact, it is most likely that both the competition (discussed in our paper) and co-operation effects work together to determine actual citation networks.

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Appendix 1 Proofs of some propositions from Section 3.1

Behavioural Assumption 1: *Any new entrant, if indifferent between r agents, investigates the earliest among them.*

Proposition 1: *In any equilibrium string, $\exists K^* \leq \frac{N-4}{2}$ s.t.$\forall k \leq K^*$,the k^{th} entry is NI and $\forall k > K^*$, the k^{th} entry is I. The exact value of K^* depends on the parameter values v, δ, p_0, w.*

Proof. We will prove this in three steps. First, let us number the agents from the end,i.e. the last agent is number 1. Let, i be the position(from the end) of the first entry of NI in a string i.e. no agent $j < i$ chooses NI. Also, let there be k_I entries of I and k_{NI} entries of NI after $i \Rightarrow k_I + k_{NI} = N - i$.

Step 1: We show that number of entries NI $\leq$ number of I's in an equilibrium string. Note that k_I agents among $N - i$ are investigating. Imposition of A1 restricts us to pure strategies$\Rightarrow$ k_I agents among $N - i$ are investigated. $\Rightarrow$ k_{NI} agents are not investigated. Now consider the decision of i. He knows that k_{NI} agents are not yet investigated. Given A1, this implies that the next k_{NI} agents will not choose to read i. He can only hope to get investigated by the agent numbered $(i - 1 - k_{NI})$, that too, conditional on the fact that all investigations done by agents $i - 1$ to $i - k_{NI}$ are failures. Now by definition of i, he is the first(from end) to choose NI. i.e. agent $i - 1, i - 2, ..., 2, 1$ all chose I. The no-deviation condition for i implies that v must be lower than his expected future payoff(A). Expected future payoff from NI =

$$A = (1 - p_0)^{k_{NI}} p_0(\delta^{k_{NI}+1}w + \delta^{k_{NI}+2}w + ... + \delta^{i-1}w) \tag{1'}$$

Payoff from deviating to I =

$$p_0 v + (1 - p_0)A \quad - c \tag{2'}$$

No profitable deviation from NI requires

$$v - \frac{c}{p_0} < A = (1 - p_0)^{k_{NI}} p_0(\delta^{k_{NI}+1}w + \delta^{k_{NI}+2}w + ... + \delta^{i-1}w) \tag{3'}$$

The first term in A is for the condition that investigation by agents $i - 1, ...1 - k_{NI}$ are failures so that i is investigated, which happens with probability $(1 - p_0)^{k_{NI}}$. Recall that the quantity p_0 is the probability that i is found useful. Also, $\delta^{k_{NI}}(\delta w + ... + \delta^{i-1-k_{NI}}w)$ is the discounted sum of payoffs if found useful. Now, note that if there are fewer than $k_{NI} + 1$ agents following i, then i is never investigated and hence i deviates from NI. But by definition i is the first to choose NI in equilibrium. This implies

$$i - 1 \geq k_{NI} + 1 \tag{4'}$$

Total number of agents $= N = (N-i)+i = (k_I+k_{NI})+1+(i-1) = (k_{NI}+1)+(k_I+i-1)$. Therefore the number of agents choosing NI$= k_{NI}$ before i and i himself $= k_{NI}+1$ and the number of agents choosing I=$k_I + i - 1 \geq i-1 \geq k_{NI}+1$ (by equation 4'). Hence in any equilibrium string, No. of I entries $\geq$ No. of NI entries.

Step 2: Now, we will prove that in fact, $i \geq k_{NI}+4$. The total number of NI$= k_{NI}+1 = k, say$.The total number of I$= N-k$ of which k are at the end. From equation (4') , $i \geq k_{NI}+2 = k+1$. Let $i = k+1$. He hopes to get investigated only by the last agent . His expected future payoff from NI is $(1-p_0)^{k-1}p_0\delta^k w < w < v - \frac{c}{p_0}$ i.e. i cannot choose NI which implies the $k+1^{th}$ entry in the equilibrium string is a I .

Now, let $i = k+2$. The next $k-1$ out of $k+1$ agents would not choose i due to A1. They would choose to investigate some agent $j > i$.Therefore i can get investigated by the second last agent. Hence, his expected payoff is $(1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) = A'$.For $i = k+2$, we need $A' > v - \frac{c}{p_0}, or\ (1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) > v - \frac{c}{p_0} > w$. So, the necessary condition for such a case to exist is $(1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) > w$ or $(1-p_0)^{k-1}p_0\delta^k\,(1+\delta) > 1$ or $(1-p_0)^{k-1}p_0 > \frac{1}{\delta^k(1+\delta)}$. For this to be satisfied, we need $(1-p_0)^{k-1}p_0 \geq \frac{1}{2}$. To see the last condition note that $\frac{1}{\delta^k(1+\delta)}$ is decreasing in δ and reaches a minimum at $1/2$ whereas the maximum value of $(1-p_0)^{k-1}p_0 < \frac{1}{4}$. Therefore $v - \frac{c}{p_0} < A'$ is not possible $\Rightarrow i > k+2$. Now, we check the same condition for $i = k+3$ and $i = k+4$. The necessary conditions for these to happen are $(1-p_0)^{k-1}p_0 \geq \frac{1}{3}$ and $(1-p_0)^{k-1}p_0 \geq \frac{1}{4}$ respectively, neither of which is possible . Hence, $i \geq k+5$.

Step 3: Suppose there exists an equilibrium string with gaps and with k agents out of N choosing NI. Define i as before ,i.e. the first agent (from the end) choosing NI. Let the agent $i+1$ choose I. Given s_{-i}, the condition for no-deviation for i from NI is

$$v - \frac{c}{p_0} < (1-p_0)^{k-1}p_0\delta^k w(1+\delta+...+\delta^{i-k-1}).$$

Similarly given $s_{-(i+1)}$ the no-deviation condition for $i+1$ is

$$v - \frac{c}{p_0} \geq (1-p_0)^{k-2}p_0\delta^k w(1+\delta+...+\delta^{i-k})$$

So the necessary condition for the two to hold simultaneously is

$$(1-p_0)^{k-2}p_0\delta^k w(1+\delta+...+\delta^{i-k}) < (1-p_0)^{k-1}p_0\delta^k w(1+\delta+...+\delta^{i-k-1})$$

or

$$(1+\delta+...+\delta^{i-k}) < (1-p_0)(1+\delta+...+\delta^{i-k-1}) \tag{5'}$$

But
$$(1-p_0)(1+\delta+...+\delta^{i-k-1}) < (1+\delta+...+\delta^{i-k-1}) < (1+\delta+...+\delta^{i-k})$$

So, (5') cannot hold. So, given the definition of i, $i+1$ must also choose NI. We can apply the same logic to any $j > i$ choosing I and will arrive at a contradiction. Reversing the numbering of agents, we conclude that there cannot be any gaps i.e. $\exists K^*$ such that $\forall j \leq K^*$ choose NI and $\forall j > K^*$ choose I. So, the number of agents choosing NI is K^* and choosing I is $N - K^*$. We also know that at least the last $K^* + 4$ agents have to choose I. So, $N - K^* \geq K^* + 4 \Rightarrow K^* \leq \frac{N-4}{2}$. The exact value of K^* is given by the following condition:

$$(1-p_0)^{K^*} p_0 \delta^{K^*} w(1+\delta+...+\delta^{N-2K^*-1}) \leq v - \frac{c}{p_0} < (1-p_0)^{K^*-1} p_0 \delta^{K^*} w(1+\delta+...+\delta^{N-2K^*}) \tag{6'}$$

■

Behavioural Assumption 2: *Any new entrant, if indifferent between r agents, investigates the most recent among them.*

Proposition 2: *In any equilibrium string,* $\exists \overline{K}, s.t. \forall i \leq \overline{K}, \forall j \leq \overline{K} - 1$, i: $NI \Rightarrow i+1 : I$ *and* $j : I \Rightarrow j+1 : NI$ *and* $\forall i > \overline{K}$, $i : I$ *.The value of* $\overline{K}$ *depends on parameter values and, for fixed* w, p_0, δ*, is decreasing in* v*.*

Proof. First note that there cannot be 2 or more consecutive NI in any equilibrium string. Suppose not. Let i and $i+1$ both choose NI, with $i+2$ choosing I. Then $i+2$ is indifferent between investigating i and $i+1$. By BA2, he chooses $i+1$. This implies that i has future payoff of zero no matter what he does. Hence, i will deviate from NI.

Next we will show that $\nexists$ 2 consecutive I entries preceded and followed by NI in the array of the equilibrium string, i.e. $\nexists$ a sequence $i, i+1, i+2, i+3$ such that i and $i+3$ choose NI and $i+1, i+2$ choose I. By way of contradiction, suppose there is. Since $i, i+3$ chooses NI, by the first argument, $i-1$ and $i+4$ choose I in equilibrium. We will now put down the no-deviation conditions for each of the agents i to $i+3$.

$i \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_1 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i})$

$i+1 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq B = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-1})$

$i+2 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq 0$

$i+3 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_3 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-3})$

Note that $A_3 < A_1$. Hence the condition required is $B \leq v - \frac{c}{p_0} \leq A_3$. which is impossible since $B > A_3$. We can conclude that in equilibrium if $\exists$ *some* i, s.t. $i, i+1$ choose I,then $s^*_k = I \ \forall k > i+1$. Otherwise, the string has to be characterised by alternating patterns ie. if $i \rightarrow NI$, then $i+1 \rightarrow I$ and if any $j \rightarrow I$, then $j+1 \rightarrow NI$. More generally, $\exists \overline{K}$ such that $\forall i < \overline{K}, \forall j < \overline{K} - 1, i \rightarrow NI \Rightarrow i+1 \rightarrow I$ and $j \rightarrow$

$I \Rightarrow j \rightarrow NI$ and $\forall i \geq \overline{K}, i \rightarrow I$. The no-deviation condition for each agent is as follows: $i \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_1 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i})$
$1 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq B = 0$
$i{+}2 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_2 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-2})$
$3 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq 0$
$4 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_4 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-4})$ ...and so on. The lower v, more of these conditions are satisfied i.e. $v - \frac{c}{p_0} < A_y$ is true for higher values of y(since the sequence $A_{1,} A_2 ...$ is decreasing). Hence the alternating pattern can go on for longer and $\overline{K}$ is higher. ∎

Behavioural Assumption 3: *Any new entrant, if indifferent between r agents, investigates them with equal probability $\frac{1}{r}$.*

Proposition 3: *In any equilibrium string, $\exists \widetilde{K}, s.t. \forall k < \widetilde{K}$ the k^{th} entry is NI and $\forall k \geq \widetilde{K}$, the k^{th} entry is I . In fact, $\widetilde{K} = 2$.*

Proof. First note that if any two entries $i, i+1$ are I, with $i-1$ being NI, then only i mixes. Player $i+1$ uses pure strategy of I_i, since the belief about all other past entrants' usefulness is less than p_0(by Lemma 1). Now, we characterise the pattern in equilibrium.

Let the $i_1^{th}, i_2^{th}, ...i_k^{th}, i_{k+1}^{th}, i_{k+2}^{th}, ...N^{th}$ agents be the ones choosing I in equilibrium; $i_1 < i_2 < .. < i_k$. Hence by definition, i_1 is the first one to choose I and everyone after agent i_k investigates some agent. We know that the last two agents would always choose I . Hence the agent i_k can be N-1 or smaller. Here, there are $i_1 - 1$ agents before i_1 who have not been investigated.Agent i_1 is indifferent between them and reads each of their papers with equal probability, $\frac{1}{i_1 - 1}$. Similarly, i_2 investigates each of $i_1, i_1{+}1, ..., i_2{-}1$ with probability $\frac{1}{i_2 - i_1}$ and so on. Given this equilibrium, we can derive the updated beliefs of each agent whenever the state of no citations is reached and hence calculate the no-deviation (unilateral) condition for each agent.

$1 : 0 < \frac{1}{i_1 - 1} p_0 w \delta^{i_1 - 1}(1 + \delta + \delta^2 + ... \delta^{N - i_1})$
$2 : v - \frac{c}{p_0} < \frac{1}{i_1 - 1} p_0 w \delta^{i_1 - 2}(1 + \delta + \delta^2 + ... \delta^{N - i_1})$
.
.
$i_1 - 1 : v - \frac{c}{p_0} < \frac{1}{i_1 - 1} p_0 w \delta(1 + \delta + \delta^2 + ... \delta^{N - i_1})$
$i_1 : v - \frac{c}{p_0} \geq \frac{1}{i_2 - i_1} p_0 w \delta^{i_2 - i_1}(1 + \delta + \delta^2 + ... + \delta^{N - i_2})$
$i_1 + 1 : v - \frac{c}{p_0} < \frac{1}{i_2 - i_1} p_0 w \delta^{i_2 - i_1 - 1}(1 + \delta + \delta^2 + ... + \delta^{N - i_2})$
.
.

.
$i_k - 1 : v - \frac{c}{p_0} < \frac{1}{i_k - i_{k-1}} p_0 w \delta (1 + \delta + ... \delta^{N-i_k})$
$i_k : v - \frac{c}{p_0} \geq p_0 w \delta (1 + \delta + ... + \delta^{N-i_k-1})$
$i_k + 1 : v - \frac{c}{p_0} \geq p_0 w \delta (1 + \delta + ... + \delta^{N-i_k-2})$
.
.
$N : v > 0$
Take the 2 equations for $i_k - 1$ and i_k.

$$p_0 w \delta (1 + \delta + ... + \delta^{N-i_k-1}) \leq v - \frac{c}{p_0} < \frac{1}{i_k - i_{k-1}} p_0 w \delta (1 + \delta + ... \delta^{N-i_k})$$

A necessary condition for this to hold is

$$p_0 w \delta (1 + \delta + ... + \delta^{N-i_k-1}) < \frac{1}{i_k - i_{k-1}} p_0 w \delta (1 + \delta + ... \delta^{N-i_k})$$

$$or, \ (1 + \delta + ... + \delta^{N-i_k-1}) < \frac{1}{i_k - i_{k-1}} (1 + \delta + ... \delta^{N-i_k})$$

Call the LHS, A. Then the previous expressions can be rewritten as:

$$A(i_k - i_{k-1}) < A + \delta^{N-i_k}$$

$$A(i_k - i_{k-1} - 1) < \delta^{N-i_k}$$

But $A > \delta^{N-i_k}$. So this can hold only if

$$i_k - i_{k-1} = 1$$

,which again implies that there is no gap between i_k and i_{k-1}.[25] Hence everyone after agent i_{k-1} investigates.

Similarly we can write out the new set of conditions where agents $i_1, i_2, ..., i_{k-2}, i_{k-1}, i_{k-1} + 1, i_{k-1} + 2, ..., N$ choose I and compare the conditions for agents i_{k-1} and i_{k-2}. We would arrive at a contradiction if $i_{k-1} - i_{k-2} > 1$. Hence in any equilibrium string there cannot be gaps. Such a string must be of the form $[NI, NI, NI.....NI, I, I, I....I]$, where the I starts at period $\widetilde{K}$.

Now we can go on further and find the value of $\widetilde{K}$. We know that all the agents before $\widetilde{K}$ were not investigated. Hence $\widetilde{K}$ investigates each of them with probability $\frac{1}{\widetilde{K}-1}$.If his

[25] Note the difference between i_{k-1} and $i_k - 1$.

investigation is not useful, $\widetilde{K}+1$ investigates $\widetilde{K}$. The no-deviation conditions for agents $\widetilde{K}-1$ and $\widetilde{K}$ are

$$p_0\delta w(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}) \leq v-\frac{c}{p_0} < \frac{1}{\widetilde{K}-1}p_0\delta w(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}})$$

The necessary condition again is

$$(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}) < \frac{1}{\widetilde{K}-1}(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}})$$

$$or,\ \widetilde{K}-1 < \frac{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}}}{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}} = 1+\frac{\delta^{N-\widetilde{K}}}{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}} < 2$$
$$i.e.\ \widetilde{K} < 3$$

Since the first agent has no one to investigate his only choice is NI. So, $\widetilde{K}<3$ implies that investigation would start from agent 2 and no later. ■

Proposition 4: *For any given set of parameters,* (v,p_0,w,δ)*,* $\widetilde{K} \leq K^* < \overline{K}$.

Proof. From Proposition 3, we know that $\widetilde{K}=2$. By way of contradiction, we assume $K^* \geq \overline{K}$ and show that set of values of the parameters that satisfy this inequality is empty.

>From Proposition 1, we know that given a K^*, the parameters should satisfy equation (6') [See Appendix].

$$\begin{aligned} L_{K^*} \quad : \quad &= (1-p_0)^{K^*}p_0\delta^{K^*}w(1+\delta+...+\delta^{N-2K^*-1}) \\ \leq \quad & v-\frac{c}{p_0} < (1-p_0)^{K^*-1}p_0\delta^{K^*}w(1+\delta+...+\delta^{N-2K^*}) = H_{K^*}. \end{aligned} \tag{3}$$

>From Proposition 2, given a $\overline{K}$, the conditions to be satisfied are

$$L_{\overline{K}} := p_0\delta w(1+\delta+...+\delta^{N-2-\overline{K}}) \leq v-\frac{c}{p_0} < p_0\delta w(1+\delta+....+\delta^{N-1-\overline{K}}) = H_{\overline{K}} \tag{4}$$

Now fix the value of $K^*=Y\geq 2$. Therefore parameters satisfy (3).

Now, we want to check whether $\overline{K}$ can be $\geq Y$.

Let $\overline{K}=Y$.

$$
\begin{aligned}
H_{K^*=Y} &= (1-p_0)^{Y\ -1} p_0 \delta^Y w(1+\delta+...+\delta^{N-2Y}) \\
&< p_0 \delta^Y w(1+\delta+...+\delta^{N-2Y}) \\
&< p_0 \delta w(1+\delta+...+\delta^{N-2Y}) \\
&= p_0 \delta w(1+\delta+...+\delta^{N-Y-Y}) \\
&= p_0 \delta w(1+\delta+...+\delta^{N-Y-\overline{K}}) \\
&\leq p_0 \delta w(1+\delta+...+\delta^{N-2-\overline{K}}) = L_{\overline{K}}|_{\overline{K}=Y}
\end{aligned}
$$

Given the fixed value of $K^* = Y$, $v - \frac{c}{p_0} < H_{K^*} < L_{\overline{K}}|_{\overline{K}=Y}$. Hence $v - \frac{c}{p_0}$ does not lie in the range $[L_{\overline{K}}, H_{\overline{K}}]|_{\overline{K}=Y}$.

Hence given (3), $\overline{K.} \neq Y$.

Also note that $L_{\overline{K}}$ is decreasing in $\overline{K}$. which implies that for values of $\overline{K} < Y, H_{K^*} < L_{\overline{K}}$. and hence (3) and (4) cannot hold together. So, given that parameter values satisfy (3), which corresponds to a K^*, $\overline{K} > K^*$. ■

Appendix 2

Proposition 8: *There cannot be an equilibrium in which there exist α and $\alpha', \alpha < \alpha'$(say), such that all players with $p_i > \alpha'$ and some with $p_i < \alpha$ enter in period 1, while players with $\alpha \leq p_i \leq \alpha'$ enter in period 2 with other players entering after period 2.*[26]

Proof. Suppose, there exists $\alpha, \alpha', \alpha''$, with $\alpha > \alpha' > \alpha''$ such that $p_i > \alpha$ and $\alpha'' < p_i < \alpha'$ enter in period 1, $\alpha' < p_i < \alpha$ enter in period 2 and $p_i < \alpha''$ enter after period 2. We will show that there will be a profitable deviation for some agent. Note that, after observing the state in each period up to and including t the probabilities of usefulness for any entrant in period $s \leq t$ are updated to $\widehat{p}^s_{t+1}$.

Case I : $\widehat{p}^2_{t+1} < \widehat{p}^1_{t+1}$ In this case the agents entering after period 2 investigate and get $\widehat{p}^1_{t+1} v - c$. But if $\widehat{p}^2_{t+1} < \widehat{p}^1_{t+1}$, these agents would prefer to enter at period 1 and get a less discounted payoff

Case II: $\widehat{p}^1_{t+1} = \widehat{p}^1_{t+1}$ In this case again, agents entering after 2 would like to enter in period 2 and get $\widehat{p}^1_2 v - c$ at period 2 .

Case III: $\widehat{p}^2_{t+1} > \widehat{p}^1_{t+1}$ This is the only case when agents entering after 2 would not want to enter earlier. We consider the following three subcases:

a) All $\alpha' \leq p_i \leq \alpha$ enter at t = 2 and Investigate: If it is optimal for these agents to do so it implies that

[26] Using period 1 in the statement is without loss of generality-we can replace it by "period τ such that there has been no entry up to $\tau - 1$."

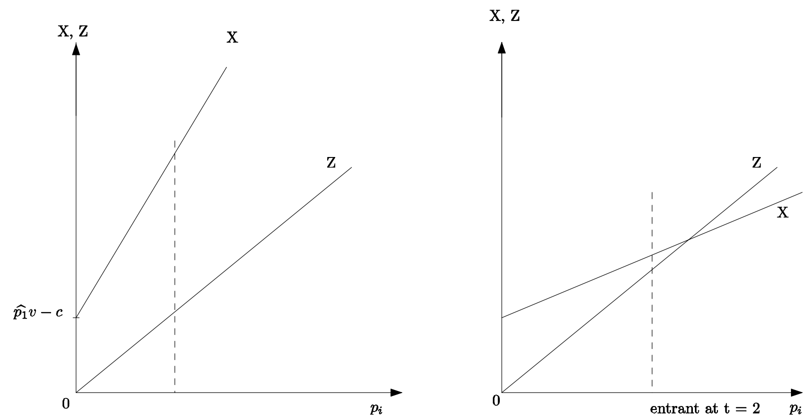

Figure 5: a b

Utility from investigating $= X = \widehat{p}_2^1 v - c + (1-\widehat{p}_2^1) p_i E_2 W_2 >$ *utility from entering at time* $1 = p_i E_1 W_1 = Z$ where $E_t W_t$ is the expected future payoff from being cited after entering in period t.[27] Agents at t=1 are potentially cited by agents at 2 and later while agents at t=2 are potentially cited by agents after 2 only. The decision is represented in Figure 5 a and 5b. Fig. 5a shows when function X is steeper than Z and 5b is the opposite. We see that in both cases if some $\alpha' \leq p_i \leq \alpha$ prefers X, it must be the case that $p_i < \alpha'$ also prefers X. Therefore, if it is optimal for agents entering at 2 to investigate and not enter in period 1, then agents $\alpha'' \leq p_i \leq \alpha'$ cannot find it profitable to enter at period 1.

b) All $\alpha' \leq p_i \leq \alpha$ enter at t $= 2$ and choose Not Investigate: In this case, agents at t>2 investigate entrants in period 2 only because of their higher probability of being useful. So, an agent entering at t=1 should deviate and wait to enter in period 2.

c) Some agents entering at t=2 investigate while some choose Not Investigate. Since later entrants are not able to tell, given no citation, whether entrants in period 2 investigated or not, there will be two revisions of probability. The probability of usefulness of period 1 entrants will drop to $\widehat{p}^{1'} < \widehat{p}^1$(from Lemma 1). The probability of usefulness of period 2 entrants will go up, i.**e.** $\widehat{p}^{2'} > \widehat{p}^2$, given that within the set of period 2 entrants the ones with higher p_i choose Not Investigate (and hence have a higher probability of not citing). Given the candidate equilibrium strategies for other players, the choice between investigating or not for players who have entered at t=2 entails comparison of $\widehat{p}_2^1 v - c + (1-\widehat{p}_2^1) p_i EW_2$, where EW' is the expected payoff from entrants at t>2 investigating second period entrants, and

[27] The inequality could be weak for a boundary type of p_i.

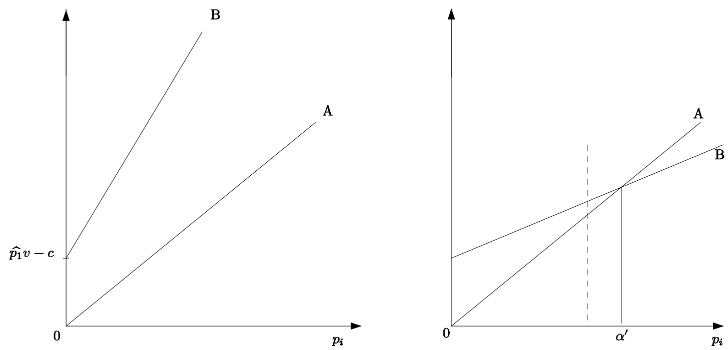

Figure 6: a b

$p_i EW_1$. This gives a cutoff p_i such that all $p_i > Y^*$ will choose NI, while others choose I. Suppose such a Y^* exists. So, we have $\alpha'' \leq p_i \leq \alpha'$ enter in period 1, $\alpha' \leq p_i \leq Y^*$ enter in period 2 and investigate. Thus there is an agent with $p_i = \alpha'$ who is indifferent between the two. The payoff from entering in period 1 $= p_i E_1 W_1$ (denoted by A) and that from entering at t=2 and investigating is $\widehat{p}_2^1 v - c + (1 - \widehat{p}_2^1) p_i EW_2$ (B). We need to compare these two payoffs as functions of p_i. Two cases are possible: i) The slope of A is less than slope of B ii) The slope of A is greater than that of B. The two cases are represented in Fig 6 a,b. In (i) the two are not equal at any p_i. So, this equilibrium is not possible. In (ii), they intersect at α'(say). Then from the graph we can see that any $p_i > \alpha'$ will prefer A to B. i.e. will prefer entering in period 1. So, for $i, \alpha' < p_i < Y^*$, entering in period 2 and investigating cannot be an equilibrium strategy. ■